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# INTERPOLATION METHODS OF I-TABLES OF ROAD LIGHTING LUMINAIRES

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# INTERPOLATION METHODS OF I-TABLES OF ROAD LIGHTING LUMINAIRES

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## Abstract

For the calculation of the photometric parameters of the road lighting it needs to have photometric data of the luminaires used for illumination of carriage ways or walkways. These data are in the form of an intensity table that provides a luminance intensity distribution emitted by the luminaire in all relevant directions. A goniophotometer is required to measure the luminous intensity distribution of luminaires. Results from goniophotometric measurements are very important for lighting engineers who are using these results in lighting calculation. Due to measurement of luminous intensity in discrete values of angle interval, interpolation methods of I-tables will be needed to enable values to be estimated for calculation of performance of road lighting system. At present is according to standard or recommendation treating with road lighting calculation recommended linear interpolation.

*Keywords*: Photometry, Interpolation, Luminous intensity

## 1 Interpolation method

Interpolation is a method of finding new values for any function using the given set of measured values. If the new value has to be found from two given points then the linear interpolation formula is used whereas if a set of three numbers are available, quadratic interpolation is appropriate and finally if a set of 'n' points are measured the new value is found using general Lagrange polynomial formula:

$$y(x) = \sum_{i=1}^{n} y_i, \ \prod_{j=1, j\neq i}^{n} \left(\frac{x-x_j}{x_i-x_j}\right) \quad (1)$$

where

- *y(x)* is the interpolated value in a point x;
- *n* is the total number of points to interpolate in between represented by the pairs of coordinates ($x_i$, $y_i$).

The most frequently used interpolating method in road lighting calculations is a linear interpolation, less often a quadratic formula is utilized. Selection of the interpolating method strongly depends on the accuracy as well as on the necessary CPU time. Note, that there is a 2-D array in all I-tables, thus an interpolation in both dimensions is required which can significantly extend a time for the mass calculations.

### 1.1 Linear interpolation

To obtain an interpolated value of the luminous intensity *I*(C,*γ)* from the I-table is neded to use measurement angles γ on the C-planes as it is defined in CIE 140. In the first step we use linear interpolation and we show the final equations for *I*(C,*γ)* calculated from four neighboring measured points(2).

$$y(x) = y_1 \left(\frac{x-x_2}{x_1-x_2}\right) + y_2 \left(\frac{x-x_1}{x_2-x_1}\right) \quad (2)$$

The coordinates $x, x_1, x_2$ are at first replaced by the azimuth angle C and then by photometric angle γ. The order of C and γ will not affect to results. If coordinates $x, x_1, x_2$ are at first replaced by the azimuth angle C then are terms $I(C,\gamma)$ and $I(C,\gamma_{j+1})$ obtain. Using this terms we obtain a searched value of the luminous intensity $I(C,\gamma)(3)$. Necessary points for linear interploation are shown on Figure 1.

$$I(C,\gamma) = I\,(C,\gamma) + \frac{\gamma-\gamma_j}{\gamma_{j+1}-\gamma_j} \cdot \left[I\left(C,\gamma_{j+1}\right) - I\left(C,\gamma_j\right)\right] \quad \text{(cd)} \tag{3}$$

## 1.2 Cubic interpolation

Linear interpolation has some loss in accuracy, generaly if larger angular intervals are used. One way to improve accuracy is to use cubic interpolation that uses more points in I table to calculate the value $I(C,\gamma)$. A comparison of the number of points needed for linear and cubic interpolation is shown in Figure 1.

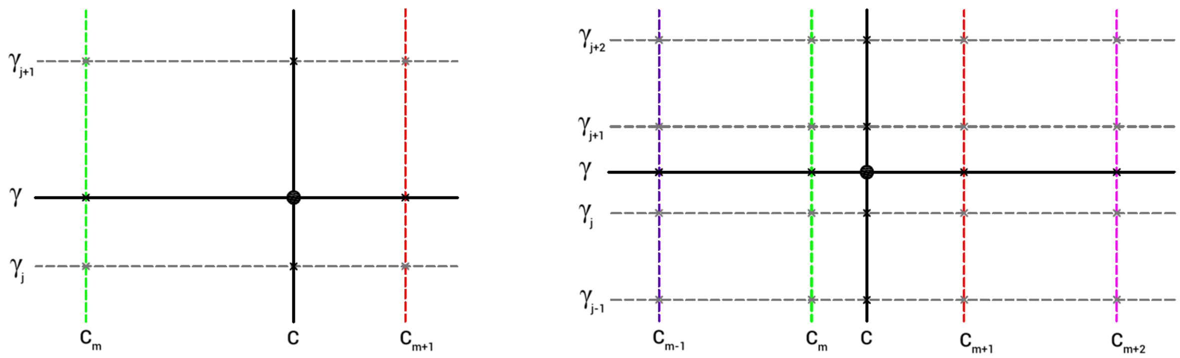



**Figure 1 – Notation of the coordinates in I-table for linear (left) and cubic (right) interpolation**

Cubic interpolation is based on following equation:

$$S_i(\mathrm{x}) = \mathrm{a_i} + \mathrm{b_i}(\mathrm{x}-\mathrm{x_i}) + \mathrm{c_i}(\mathrm{x}-\mathrm{x_i})^2 + \mathrm{d_i}(\mathrm{x}-\mathrm{x_i})^3 \tag{4}$$

The coordinates $x,x_i$ are at first replaced by the azimuth angle C and then by photometric angle γ. The order of C and γ will not affect to results. We are looking for the solution of following polynomial system(5,6,7) to find the value $I(C,\gamma_{j-1})$.

$$S_0(\mathrm{C}) = \mathrm{a_0} + \mathrm{b_0}(\mathrm{C}-\mathrm{C_{m-1}}) + \mathrm{c_0}(\mathrm{C}-\mathrm{C_{m-1}})^2 + \mathrm{d_0}(\mathrm{C}-\mathrm{C_{m-1}})^3 \tag{5}$$

$$S_1(\mathrm{C}) = \mathrm{a_1} + \mathrm{b_1}(\mathrm{C}-\mathrm{C_m}) + \mathrm{c_1}(\mathrm{C}-\mathrm{C_m})^2 + \mathrm{d_1}(\mathrm{C}-\mathrm{C_m})^3 \tag{6}$$

$$S_2(\mathrm{C}) = \mathrm{a_2} + \mathrm{b_2}(\mathrm{C}-\mathrm{C_{m+1}}) + \mathrm{c_2}(\mathrm{C}-\mathrm{C_{m+1}})^2 + \mathrm{d_2}(\mathrm{C}-\mathrm{C_{m+1}})^3 \tag{7}$$

Polynomials pass through specified points (8,9,10,11).

$$S_0(\mathrm{C_{m-1}}) = I\left(C_{m-1};\gamma_{j-1}\right) \Rightarrow \mathrm{a_0} = I\left(C_{m-1};\gamma_{j-1}\right) \tag{8}$$

$$S_1(\mathrm{C_m}) = I\left(C_m;\gamma_{j-1}\right) \Rightarrow \mathrm{a_1} = I\left(C_m;\gamma_{j-1}\right) \tag{9}$$

$$S_2(\mathrm{C_{m+1}}) = I\left(C_{m+1};\gamma_{j-1}\right) \Rightarrow \mathrm{a_2} = I\left(C_{m+1};\gamma_{j-1}\right) \tag{10}$$

$$S_2(\mathrm{C_{m+2}}) = I\left(C_{m+2};\gamma_{j-1}\right) \Rightarrow$$
$$I\left(C_{m+1};\gamma_{j-1}\right) + \mathrm{b_2}(C_{m+2}-\mathrm{C_{m+1}}) + \mathrm{c_2}(C_{m+2}-\mathrm{C_{m+1}})^2 + \mathrm{d_2}(C_{m+2}-\mathrm{C_{m+1}})^3 \tag{11}$$

Polynomials are linked on each other (12,13).

$$S_0(\mathrm{C_m}) = S_1(\mathrm{C_m}) = I\left(C_m;\gamma_{j-1}\right) = I\left(C_{m-1};\gamma_{j-1}\right) + \mathrm{b_0}(C_m - \mathrm{C_{m-1}}) + \mathrm{c_0}(C_m - \mathrm{C_{m-1}})^2 + \mathrm{d_0}(C_m - \mathrm{C_{m-1}})^3 \quad (12)$$

$$S_1(\mathrm{C_{m+1}}) = S_2(\mathrm{C_{m+1}}) = I\left(C_{m+1};\gamma_{j-1}\right) = I\left(C_m;\gamma_{j-1}\right) + \mathrm{b_0}(C_{m+1} - \mathrm{C_m}) + \mathrm{c_0}(C_{m+1} - \mathrm{C_m})^2 + \mathrm{d_0}(C_{m+1} - \mathrm{C_m})^3 \quad (13)$$

First derivatives are linked on each other (14,15):

$$\mathrm{S_0'}(\mathrm{C_m}) = \mathrm{S_1'}(\mathrm{C_m}) = \mathrm{b_0} + 2\mathrm{c_0}(C_m - \mathrm{C_{m-1}}) + 3\mathrm{d_0}(C_m - \mathrm{C_{m-1}})^2 = \mathrm{b_1} \quad (14)$$

$$\mathrm{S_1'}(\mathrm{C_{m+1}}) = \mathrm{S_2'}(\mathrm{C_{m+1}}) = \mathrm{b_1} + 2\mathrm{c_1}(C_{m+1} - \mathrm{C_m}) + 3\mathrm{d_1}(C_{m+1} - \mathrm{C_m})^2 = \mathrm{b_2} \quad (15)$$

Second derivatives are linked on each other (16,17):

$$\mathrm{S_0''}(\mathrm{C_m}) = \mathrm{S_1''}(\mathrm{C_m}) = 2\mathrm{c_0} + 6\mathrm{d_0}(C_m - \mathrm{C_{m-1}}) = 2\mathrm{c_1} \quad (16)$$

$$\mathrm{S_1''}(\mathrm{C_{m+1}}) = \mathrm{S_2''}(\mathrm{C_{m+1}}) = 2\mathrm{c_1} + 6\mathrm{d_1}(C_{m+1} - \mathrm{C_m}) = 2\mathrm{c_2} \quad (17)$$

Conditions for a natural cubic spline (18,19).

$$\mathrm{S_0''}(\mathrm{C_{m-1}}) = 0 \Rightarrow 2\mathrm{c_0} = 0 \quad (18)$$

$$\mathrm{S_2''}(\mathrm{C_{m+2}}) = 0 \Rightarrow 2\mathrm{c_2} + 6\mathrm{d_2}(C_{m+2} - \mathrm{C_{m+1}}) = 0 \quad (19)$$

For the determination of coeficient bi,ci,di are need following substitutions:

$$h_i = C_{m+i} - C_{m-1+i}\,;\, h_{i+1} = C_{m+1+i} - C_{m+i};\ \Delta y_i = \gamma_{j+i} - \gamma_{j-1+i};\, \Delta y_{i+1} = \gamma_{j+1+i} - \gamma_{j+i};$$

Coefficients $b_i$,$c_i$,$d_i$ are calculated by using following equations (20,21,22).

$$\mathrm{h_i c_i} + 2\,(\mathrm{h_{i+1}} + \mathrm{h_i})\,\mathrm{c_{i+1}} + \mathrm{h_{i+1}c_{i+2}} = 3\left(\frac{\Delta\,\mathrm{y_{i+1}}}{\mathrm{h_{i+1}}} - \frac{\Delta\,\mathrm{y_i}}{\mathrm{h_i}}\right) \quad (20)$$

$$\mathrm{b_i} = \frac{\Delta\,\mathrm{y_i}}{\mathrm{h_i}} - \frac{\mathrm{h_i}}{3}(\mathrm{c_{i+1}} + 2\mathrm{c_i}) \quad (21)$$

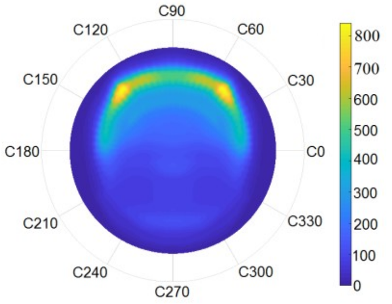
$$\mathrm{d_i} = \frac{\mathrm{c_{i+1}} - \mathrm{c_i}}{3\mathrm{h_i}} \quad (22)$$

Then is needed to repeat this procedure for angles $\gamma_j$, $\gamma_{j+1}$, $\gamma_{j+2}$. To find the result must be C replaced with angles $\gamma_{j11}$, $\gamma_j$, $\gamma_{j+1}$, $\gamma_{j+2}$ and the value of I(C, γ) passing through these angles and C is than calculated.

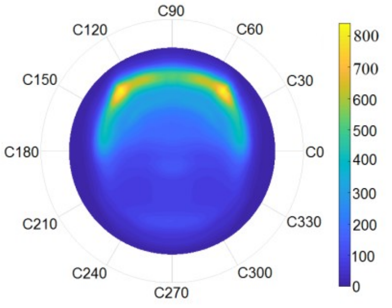
## 2 Simulation

The interpolating methods of luminous intensity data can significantly improve the accuracy of the calculations and redound to more effective and reliable road lighting design. Two luminaires were chosen to compare the accuracy of the currently used linear interpolation and cubic interpolation. To verify accuracy, values between C = 5 ° and γ = 2.5 ° or C = 15 ° and γ = 2.5 ° were calculated by interpolation (to C = 2.5 ° and γ = 0.5 °) and compared with the measured values C = 2.5 ° and γ = 0.5 °. The calculated values are shown in the following tables. Relative root mean square error (RRMSE), R-Squared ($R^2$) and total luminous flux of calculated values are shown in tables 7 and 8.

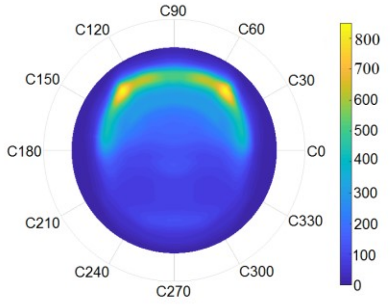
**Table 1 – Change of LIDC in hemisphere for linear and cubic interpolation (Luminaire 1 )**




Measured values C=2,5°, γ = 0,5° (values in cd/1000lm)

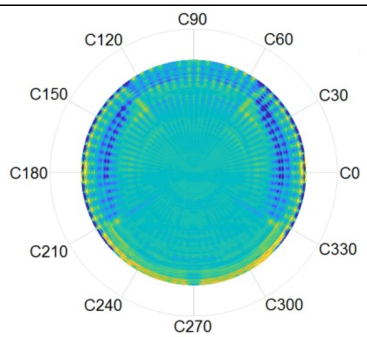



Linear Interpolation, Input C=5°, γ = 2,5°

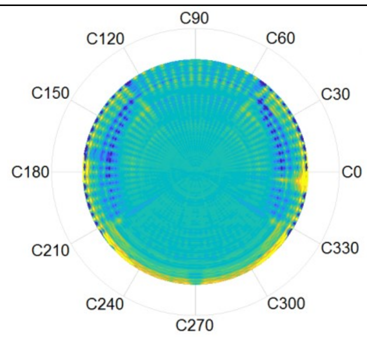



Cubic Interpolation, Input C=5°, γ = 2,5°

**Table 2 – ERR (%) for linear and cubic interpolation (Luminaire 1, Input C = 5°, γ = 2,5° )**




Linear Interpolation, Input C=5°, γ = 2,5°




Cubic Interpolation, Input C=5°, γ = 2,5°

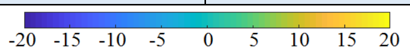
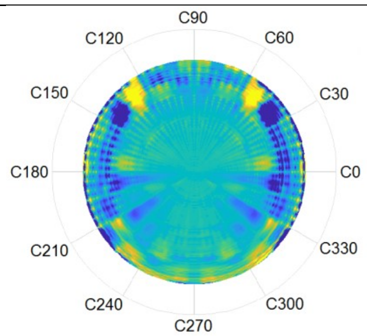
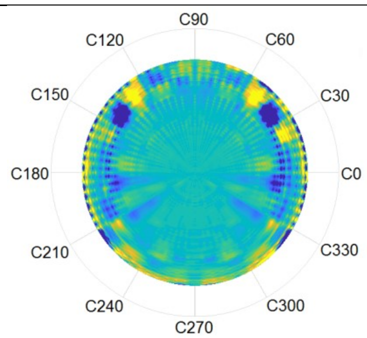



**Table 3 – ERR (%) for linear and cubic interpolation (Luminaire 1, Input C = 15°, γ = 2,5° )**




Linear Interpolation, Input C=5°, γ = 2,5°




Cubic Interpolation, Input C=5°, γ = 2,5°

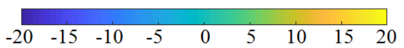
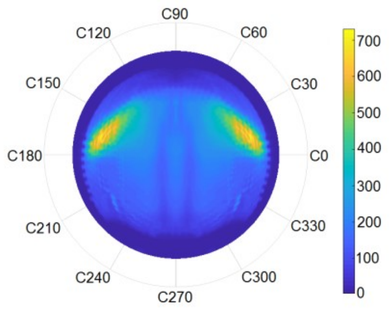
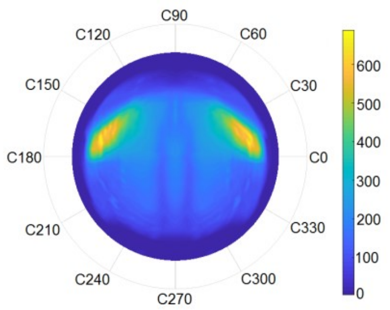
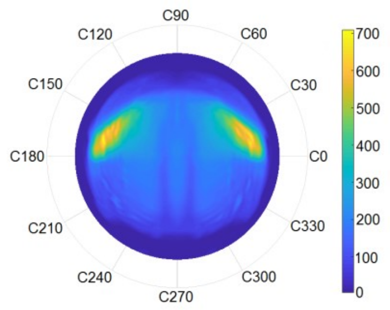


**Table 4 – Change of LIDC in hemisphere for linear and cubic interpolation (Luminaire 2 )**



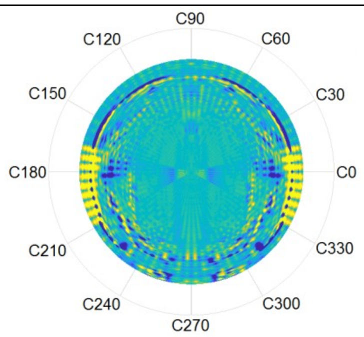

| Measured values C=2,5°, γ = 0,5° (values in cd/1000lm) | |
|---|---|

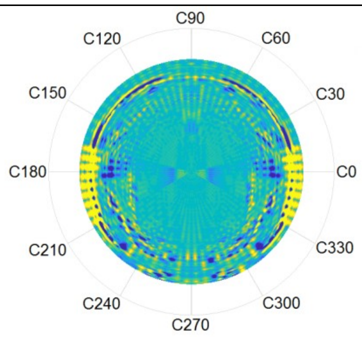



| Linear Interpolation, Input C=5°, γ = 2,5° | Cubic Interpolation, Input C=5°, γ = 2,5° |
|---|---|

**Table 5 – ERR (%) for linear and cubic interpolation (Luminaire 2, Input C = 5°, γ = 2,5° )**




| Linear Interpolation, Input C=5°, γ = 2,5° | Cubic Interpolation, Input C=5°, γ = 2,5° |
|---|---|

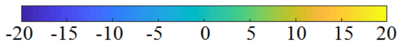
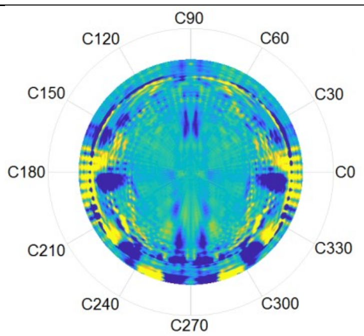
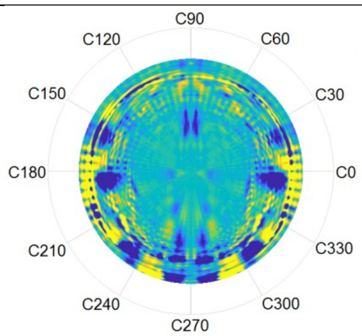



**Table 6 – ERR (%) for linear and cubic interpolation (Luminaire 2, Input C = 15°, γ = 2,5° )**




| Linear Interpolation | Cubic Interpolation |
|---|---|

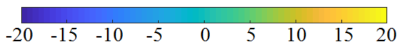


**Table 7 – Calculated values for linear and cubic interpolation ( Input C = 5º, γ = 2,5º )**

| Measured angular interval | C(°) | 5 | γ(°) | 2.5 | Φ (meas) [lm] | Φ (Calc) [lm] |
|---|---|---|---|---|---|---|
| Calculated angular interval | C(°) | 2.5 | γ(°) | 0.5 | | |
| Luminaire 1 | | | | | | |
| Linear interpolation | RRMSE (%) | 7.4565 | $R^2$ | 0.9967 | 2601.48 | 2601.78 |
| Cubic spline interpolation | RRMSE (%) | 6.9874 | $R^2$ | 0.9971 | | 2601.64 |
| Luminaire 2 | | | | | | |
| Linear interpolation | RRMSE (%) | 14.1578 | $R^2$ | 0.9846 | 4497.54 | 4461.57 |
| Cubic spline interpolation | RRMSE (%) | 14.0759 | $R^2$ | 0.9848 | | 4462.1 |

**Table 8 – Calculated values for linear and cubic interpolation ( Input C = 15º, γ = 2,5º )**

| Measured angular interval | C(°) | 15 | γ(°) | 2.5 | Φ (meas) [lm] | Φ (Calc) [lm] |
|---|---|---|---|---|---|---|
| Calculated angular interval | C(°) | 2.5 | γ(°) | 0.5 | | |
| Luminaire 1 | | | | | | |
| Linear interpolation | RRMSE (%) | 15.4077 | $R^2$ | 0.9855 | 2601.48 | 2599.08 |
| Cubic spline interpolation | RRMSE (%) | 13.9632 | $R^2$ | 0.9884 | | 2600.09 |
| Luminaire 2 | | | | | | |
| Linear interpolation | RRMSE (%) | 19.7464 | $R^2$ | 0.9688 | 4497.54 | 4494.52 |
| Cubic spline interpolation | RRMSE (%) | 18.4513 | $R^2$ | 0.9739 | | 4494.78 |

## 3 Conclusion

It can be seen that by using of cubic interpolation is possible to achieve better accuracy of calculated values as in linear interpolation. The resulting accuracy is influenced by the shape of the luminous intensity distribution curve and density of measured angular intervals To verify the accuracy of cubic interpolation, it is necessary to make simulations on the others samples. It can be said that the results presented in the paper show that cubic interpolation can be considered in future revisions of CIE 140 and other related standard. For luminaires that contain local extremes in luminous intensity distribution curve, it would be possible to consider a hybrid adaptive spline interpolation that is capable of reconstructing the given extremes.

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## References

CIE 1982. CIE 30-2:1982. Calculation and measurement of luminance and illuminance in road lighting. Vienna: CIE.

CIE 2019. CIE 140:2019. Road lighting calculations, 2nd edition. Vienna: CIE.

CEN - EN 13201-3:2015. Road lighting - Part 3: Calculation of performance. Brussels: CEN.